exchange resin which was placed on top of a 50-cm column. The column was eluted with 3.5 *p*H ammonium lactate solution.[11]

[11] W. E. Nervik, J. Phys. Chem. **59**, 690 (1955).

(9) After the samarium activities were eluted from the column, they were re-adsorbed on the top and again run through the column to insure radiochemical purity from the other rare earths and, in particular, from the yttrium activities.

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Distorted-Wave Calculations of Light Nuclei (d,p) Angular Distributions*†

WILLIAM R. SMITH AND EUGENE V. IVASH
The University of Texas, Austin, Texas
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Deuteron-stripping angular distributions have been calculated for 14 light-target reactions ($A \leq 48$) over a range of bombarding energies, using the distorted-wave Born approximation with diffuse-well optical-model nuclear potentials. A fair degree of agreement with experiment has been obtained, though in many cases the results, which depend strongly on the particular reaction considered, are inferior. In some of the latter instances the data are not well fitted; in others the agreement between elastic-scattering and stripping parameters, or between parameters for stripping leading to different residual levels of the same final nucleus, is poor. For reactions with $L_N = 0$ it appears that the angular distributions can be reasonably fitted in the neighborhood of the Coulomb barrier as the bombarding energy is raised only if the deuteron real potential depth is appreciably increased.

INTRODUCTION

In a previous paper[1] results of calculations for deuteron stripping differential cross sections based on the distorted-wave Born approximation with diffuse-well optical model nuclear potentials were presented for 14 reactions for nuclei with $A \geq 59$. The present study extends this work to 14 reactions for light nuclei with $A \leq 48$. The main purpose of the investigation, as previously, was to ascertain the degree of applicability of the distorted-wave Born approximation with optical potentials in the determination of stripping differential cross sections. In particular, it was hoped that a set of optical-model parameters having only limited and systematic variations could be found which would yield agreement, over a wide range, with experimental data for light nuclei. This search has only been partially successful. Appreciable and nonsystematic variations in the optical parameters are obtained in many cases, in contrast to the results found for most of the heavier nuclei previously studied,[1] and the consistency in the results for different reactions is generally poor. In some instances, more than one acceptable set of parameters is determined, even under conditions in which the usual VR^2 ambiguity can be excluded.

Thus, the results presented here for light nuclei are to be accepted only with a considerable degree of caution. Not only does the distorted-wave Born approximation with optical potentials seem poorer than for the heavier nuclei, but the complexity of the calculations makes it entirely possible that in many cases more extensive work will disclose the existence of parameter regions yielding appreciably better results than obtained here.[2]

CALCULATIONAL PROCEDURE

It has been shown previously[1] for heavier targets ($A \geq 59$) that (1) optical-model parameters yielding agreement with the stripping data exist which do not vary appreciably from one reaction to the next, and (2) these parameters are in close accord with those obtained from elastic-scattering data. Such consistency between various reactions, unfortunately, has not been found for light targets ($A \leq 32$). Hence, it was considered advisable to adopt an approach in the stripping calculations for light nuclei somewhat different from that used in reference 1.

Because of the lack of over-all consistency, and the VR^2 ambiguity, the potential radii have been kept fixed at certain values for all of the light nuclei reactions. The following somewhat arbitrary values based on preliminary calculations have been adopted[3]:

$$R_{0p} = 1.25 \text{ F}, \quad R_{0d} = 1.4 \text{ F},$$

* This work was supported in part by the U. S. Atomic Energy Commission.

† Based on a dissertation (W. R. Smith) submitted in partial fulfillment of the requirements for the Ph.D. degree at the University of Texas.

[1] W. R. Smith and E. V. Ivash, Phys. Rev. **128**, 1175 (1962).

[2] Approximately 3000 angular distributions for light nuclei have been obtained; however, 14 parameters are involved in the calculation, not including at least two necessary to take into account spin-orbit effects.

[3] The notation used in this article agrees with that of reference 1.

which, unless otherwise stated, have been used in the present investigation. The diffuseness parameters have also been fixed at the values

$$a_p=0.5 \text{ F}, \quad a_d=0.7 \text{ F}.$$

However, in many cases a_p and a_d were varied after determining the potential depths in order to see if improved agreement could be obtained. Finally, Woods-Saxon real and imaginary potentials have been employed.[4]

The bound-state neutron radial wave function in each case has the number of nodes predicted by the shell model. It is obtained by numerical solution of the radial Schrödinger equation using a Woods-Saxon potential form factor. For simplicity, the neutron and real deuteron form factors are the same in all cases presented here.

Even though a set of parameters approximately constant could not be found for all of the light nuclei reactions considered, it was discovered that the reactions could be separated into groups so that the results within a group were similar. In an effort to secure more uniformity many cross checks were made to see whether parameters found for one reaction could be used in other cases.

In the following sections the reactions studied are grouped according to L_N value, since, as will be seen, there are certain similarities in the results for a given L_N. The order within an L_N group corresponds approximately to the sequence in which the reactions were considered. The pertinent properties of the reactions studied are listed in Table I in the order of increasing atomic weight of the target nucleus. All of the stripping

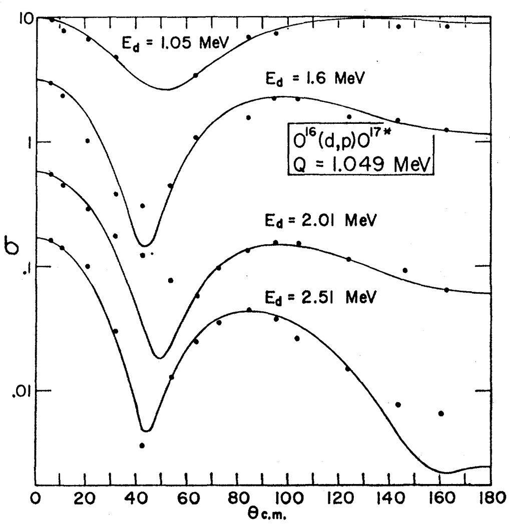

FIG. 1. Comparison of experimental and theoretical angular distributions for the $O^{16}(d,p)O^{17*}$, $L_N=0$ reaction for $E_d=1.05$, 1.6, 2.01, and 2.51 MeV. The parameters are listed in Table II.

[4] Thus, the form factors used here differ somewhat from those of reference 1.

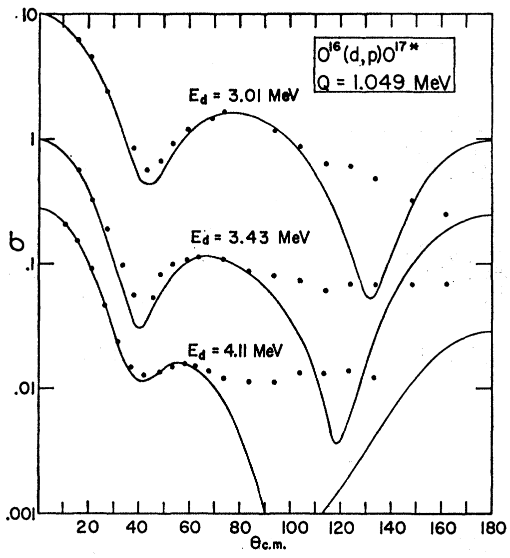

FIG. 2. Comparison of experimental and theoretical angular distributions for the $O^{16}(d,p)O^{17*}$, $L_N=0$ reaction for $E_d=3.01$, 3.43, and 4.11 MeV. The parameters are listed in Table II.

angular distributions are plotted with arbitrary normalization.

RESULTS FOR $L_N=0$ REACTIONS

$O^{16}(d,p)O^{17*}$ (0.875 MeV)

We begin the discussion of the results obtained by considering the $O^{16}(d,p)O^{17*}$ 0.875-MeV level, $L_N=0$ reaction, the most extensively studied of the reactions treated in the present investigation.[5] O^{16} is expected to be a favorable theoretical case since its doubly closed shell structure should allow the interaction between the captured neutron and the nucleus to be well approximated by a simple central potential.

The relative differential cross sections obtained are presented in Figs. 1, 2, and 3 for a range of bombarding energies E_d from 1.05 to 15 MeV. Table II lists the best-fit optical-model parameters determined for the various E_d. Perhaps the most striking feature of the results is the considerable increase required in V_d as E_d is raised from 2 to 15 MeV, the rate of increase being maximum in the region of the Coulomb barrier ($E_d \approx 3$ MeV). If V_d is kept constant at its $E_d=2$-MeV value, then the calculated positions of the second peak for energies between 2.51 and 4.11 MeV are found to be displaced in the backward direction with respect to the experimental results. Figure 4 shows the angular distributions obtained when values of V_d smaller than the optimum are used for the $E_d=7.73$-MeV reaction. A similar variation in V_d is observed for other choices of the radii —in particular, for $R_{0p}=1.31$ F, $R_{0d}=1.51$ F, and

[5] Also studied by J. L. Richter and E. V. Ivash, Phys. Rev. **111**, 245 (1958), although the curves based on the optical model are in error.

Table I. List of reactions.

Reaction	Residual energy (MeV)	Neutron orbital	E_d (MeV)	Q (MeV)	Figure	Reference
$Be^9(d,p)Be^{10}$	0	$1p_{3/2}$	3.6	4.585	19	a
$B^{10}(d,p)B^{11}$	0	$1p_{3/2}$	8.2	9.24	20	b
			15.5		20	c
			21.5		20	c
			28		20	d
$C^{12}(d,p)C^{13}$	0	$1p_{1/2}$	9	2.719	19, 24	e
$C^{12}(d,n)N^{13}$	0	$1p_{1/2}$	2.75	−0.286	19	f
$C^{12}(d,p)C^{13*}$	3.09	$2s_{1/2}$	2.889	−0.371	12	g
			9		12	e
			13.3			h
$O^{16}(d,p)O^{17}$	0	$1d_{5/2}$	0.58	1.919	13	i
			1.05		13	j
			2.01		13	j
			2.65		17	k
			3.01		13, 16	k
			3.49			l
			4.11		14	l
			7.73		14, 15, 16, 22, 23	m
			15		14	n
$O^{16}(d,p)O^{17*}$	0.875	$2s_{1/2}$	1.05	1.049	1	j
			1.6		1	j
			2.01		1	j
			2.51		1, 5, 6	j
			3.01		2	k
			3.43		2, 5	k
			4.11		2	l
			7.73		3, 4, 22, 23	m
			15		3, 5	n
$Mg^{24}(d,p)Mg^{25}$	0	$1d_{5/2}$	10	5.107	18	o
			14.8		18	p
$Mg^{24}(d,p)Mg^{25*}$	0.58	$2s_{1/2}$	3.9	4.519	11	q
			10		11	o
$Si^{28}(d,p)Si^{29}$	0	$2s_{1/2}$	4	6.249	7, 9	r
			6.2		7, 9	s
			8		7, 10	s
			10		8	t
			15		8	u
$Si^{28}(d,p)Si^{29*}$	1.28	$1d_{3/2}$	8	4.969		s
$S^{32}(d,p)S^{33}$	0	$1d_{3/2}$	4	6.421	18	r
$Ca^{48}(d,p)Ca^{49}$	0	$2p_{3/2}$	7	2.921	21	v
$Ca^{48}(d,p)Ca^{49*}$	2.026	$2p_{1/2}$	7	0.895	21	v

[a] H. W. Fulbright, J. A. Bruner, D. A. Bromley, and L. M. Goldman, Phys. Rev. **88**, 700 (1952).
[b] B. Zeidman and J. M. Fowler, Phys. Rev. **112**, 2020 (1958).
[c] See reference 15.
[d] R. J. Slobodrian, Phys. Rev. **126**, 1059 (1962).
[e] J. A. Kuehner, E. Almqvist, and J. E. Evans (private communication).
[f] A. Elwyn, J. V. Kane, S. Ofer, and D. H. Wilkinson, Phys. Rev. **116**, 1490 (1959).
[g] See reference 13.
[h] N. I. Zaika, O. F. Nemets, and M. A. Tserineo, Zh. Eksperim. i Teor. Fiz. **39**, 3 (1960) [translation: Soviet Phys.—JETP **12**, 1 (1961)].
[i] Mira K. Juric, Phys. Rev. **98**, 85 (1955).
[j] See reference 9.
[k] T. F. Stratton, J. M. Blair, K. F. Famularo, and R. V. Stuart, Phys. Rev. **98**, 629 (1955).
[l] See reference 10.
[m] E. J. Burge, H. B. Burrows, W. M. Gibson, and J. Rotblat, Proc. Roy. Soc. (London) **A210**, 534 (1952).
[n] E. L. Keller, Phys. Rev. **121**, 820 (1961).
[o] S. Hinds and R. Middleton, in *Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961*, edited by J. B. Birks (Heywood and Company Ltd., London, 1961), p. 446.
[p] E. W. Hamburger and A. G. Blair, Phys. Rev. **119**, 777 (1960).
[q] S. A. Cox and R. M. Williamson, Phys. Rev. **105**, 1799 (1957).
[r] I. B. Teplov and B. A. Iur'ev, Zh. Eksperim. i Teor. Fiz. **34**, 334 (1958) [translation: Soviet Phys.—JETP **7**, 233 (1958)].
[s] J. A. Kuehner, E. Almqvist, and D. A. Bromley, Nucl. Phys. **21**, 555 (1960).
[t] R. Bercaw (private communication).
[u] A. G. Blair (private communication).
[v] E. Kashy, A. Sperduto, H. A. Enge, and W. W. Buechner, Bull. Am. Phys. Soc. **7**, 315 (1962).

$R_{0p}=1.3$ F, $R_{0d}=1.3$ F—as well as for the $L_N=0$ reactions $C^{12}(d,p)C^{13*}$, $Mg^{24}(d,p)Mg^{25*}$, and $Si^{28}(d,p)Si^{29}$.

A second set of values for V_d, higher than the first, is found which yields results nearly as satisfactory, the other parameters remaining unchanged except for the imaginary potentials which are somewhat reduced. Typical examples are shown in Fig. 5. A similar higher set for V_d is obtained for other stripping reactions, as well as for elastic scattering.[6] Presumably a corresponding ambiguity with respect to V_p also exists. These higher values appear to be nonphysical, but complicate the decision as to whether a given set of "best-fit" parameters is, indeed, the correct choice. It is interesting to note that for the higher set, as for the lower one, V_d

[6] E. C. Halbert, R. H. Bassel, and G. R. Satchler, Bull. Am. Phys. Soc. **7**, 357 (1962).

TABLE II. Best-fit parameters.

Reaction	E_d (MeV)	L_N	V_p (MeV)	V_d (MeV)	W_p (MeV)	W_d (MeV)	Calc. σ_{abs} (mb/sr)	$\sigma_{calc}/\sigma_{exp}$	Fig.
$Be^9(d,p)Be^{10}$	3.6	1	60	80	3	6	0.86	0.276	19
$B^{10}(d,p)B^{11}$ [a]	8.1	1	53	74	5	10	2.14		20
	15.5		50	73	12	24	1.25		20
	21.5		48	78	14	24	1.03		20
	28		45	80	20	20	0.92	0.269	20
$C^{12}(d,n)N^{13}$	2.75	1	51	80	2	4	17.45		19
$C^{12}(d,p)C^{13}$ [a]	9	1	46	87	2	4	12.78	0.835	19
$C^{12}(d,p)C^{13*}$	2.889	0	75	75	4	8	114.6	0.686	12
	9		75	107	9	18	107.8	0.352	12
$O^{16}(d,p)O^{17}$	0.58	2	66	73.5	0.5	1	3.57		13
	1.05		66	72.5	1.5	3	8.88	3.7	13
	2.01		66	73	0.6	1.2	51.41	7.34	13
	2.01		66	73	3	6	18.21	2.6	13
	3.01		66	72.5	2	6.7	29.44	0.892	13
	4.11		66	73	6	10	21.65	0.618	14
	7.73		64	76	8	13	21.98	0.733	14
	15		65	80	6	12	28.45	0.913	14
	3.01		66	115	1.5	3	23.0	0.697	16
	7.73		64	122	3	6	16.42	0.547	16
$O^{16}(d,p)O^{17*}$	1.05	0	64	76	5	10	6.71	0.745	1
	1.6		64	76	3	5.2	24.72	1.03	1
	2.01		64	75.5	5	9	71.61	0.682	1
	2.51		64	78.5	4	6.3	101.0	0.842	1
	3.01		64	85	4	4	171.8	0.859	2
	3.43		64	88	3	3	142.0	0.596	2
	4.11		64	92	5	5	153.9	0.515	2
	7.73		63	100	7	16	152.4	0.328	3
	15		56	107	4	8	46.3	0.396	3
	15		51	75	4	8	58.5	0.5	3
	2.51		66	122	2.05	4.1	71.26	0.594	5
	3.43		66	136	2.2	2.2	164.5	0.503	5
	15		56	170	3	6	37.89	0.324	5
$Mg^{24}(d,p)Mg^{25}$	10	2	55	80	5	10	15.9		18
	10		35	100	1	7	10.19		18
	14.8		47	60	2	7	33.2	10.06	18
$Mg^{24}(d,p)Mg^{25*}$	3.9	0	50	76	2	4	49.56		11
	10		50	96	4	12.3	80.15		11
$Si^{28}(d,p)Si^{29}$	4	0	53	68	6	8	32.0		7
	6.2		51	98	3	8	57.72		7
	8		50	95	2	8	66.9	1.454	7
	10		46	96	2	6	54.94		8
	15		47	101	2	7	44.08	1.633	8
	15		40.5	67	10	13	53.08	1.944	8
$S^{32}(d,p)S^{33}$	4	2	58	60	2	2	18.86		18
$Ca^{48}(d,p)Ca^{49}$	7	1	55	63	8	15	40.33	0.577	21
$Ca^{48}(d,p)Ca^{49*}$	7	1	55	63	8	15	23.0	0.432	21

[a] For $B^{10}(d,p)B^{11}$ and $C^{12}(d,p)C^{13}$, $a_p=0.4$ F and $a_d=0.6$ F.

increases as the bombarding energy is raised, the ratio $V_d(E_d)/V_d(E_d=15\text{ MeV})$ being approximately the same for the two sets of V_d's.

To further complicate matters, a third fit to the $E_d=15$ MeV data (Fig. 3) is obtained using a value for V_d in agreement with the results found for bombarding energies below 2.5 MeV. However, the angular distribution for this case differs considerably from the previous distributions at back angles, so that measurements of the cross section beyond 90° should determine which choice of parameters is best.

Because the fits become rapidly worse at angles beyond the second peak as the bombarding energy is raised from 3 to about 4 MeV, effects ignored in the usual distorted-wave Born approximation treatment presumably become important in this region, and the results obtained for the optical-model parameters must be viewed with some reservation. It is possible that this poor agreement is connected with the variation in V_d also observed in this region.

It is interesting to speculate as to the origin of this variation. One possible mechanism is related to the Oppenheimer-Phillips process.[7] As the energy of the incoming deuteron is progressively lowered below the Coulomb barrier, it becomes more and more difficult for the proton to reach the surface of the nucleus, the effective interaction of the proton with the nucleus is diminished, and the effective interaction of the deuteron with the nucleus, described by V_d, is decreased. Such an effect should be largest for $L_N=0$ reactions, for which

[7] J. R. Oppenheimer and M. Phillips, Phys. Rev. 48, 500 (1935).

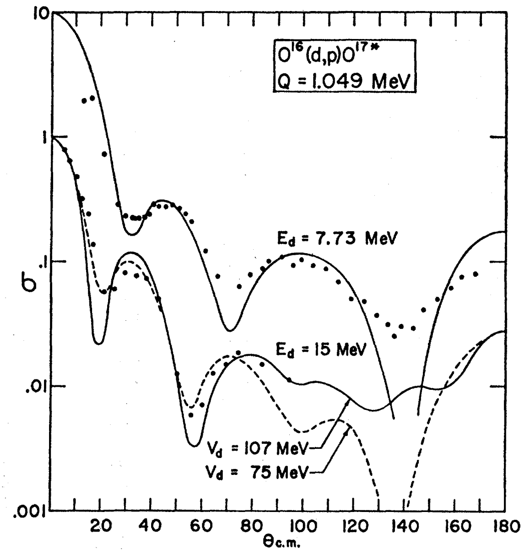

FIG. 3. Comparison of experimental and theoretical angular distributions for the $O^{16}(d,p)O^{17*}$, $L_N=0$ reaction for $E_d=7.73$ and 15 MeV. The parameters are listed in Table II.

the average deuteron impact parameter has its minimum value. However, this explanation does not seem to account for the apparent absence of a similar variation in V_d for the heavier nuclei.[1]

An attempt was also made to determine V_d from deuteron elastic-scattering data for oxygen (Table III).

TABLE III. Elastic-scattering parameters.

Reaction	E (MeV)	R_0 (F)	a (F)	V (MeV)	W (MeV)	V'^{a} (MeV)	Reference
$C(p,p)C$	14.0	1.24	0.51	49.2	8.5	48.5	b
	19.4	1.24	0.54	48.9	8.0	48.1	b
$N(p,p)N$	10	1.2	0.6	49	3.0	45.1	c
$Al(p,p)Al$	9.8	1.45	0.19	40.4	9.2	54.4	d
	17.6	1.29	0.48	51.8	8.6	55.4	b
$A(p,p)A$	9.72	1.2	0.41	62	9.5	57.1	d
$O(d,d)O$	11.2	1.4	0.661	58.5	18.7	58.5	e, f
		1.4	0.607	113.4	20	113.4	
$Mg(d,d)Mg$	4.07	1.4	0.625	56.7	15.4	56.7	e, g
		1.4	0.604	95.0	17.3	95.0	
	10.1	1.5	0.55	83	27.8	95.3	h
	11.8	1.4	0.667	53.6	22.5	53.6	e, i
		1.4	0.684	107.2	25.2	107.2	
$Al(d,d)Al$	10.1	1.4	0.668	72.7	47.0	72.7	e, j
		1.4	0.636	98.6	43.4	98.6	
	15	1.5	0.6	55	25	63.1	k
$Ti(d,d)Ti$	15	1.5	0.6	59	21	67.7	k

[a] The value of V corresponding to $R_{0p}=1.25$ F or $R_{0d}=1.4$ F is calculated using the approximate relation VR^2 =const.
[b] A. E. Glassgold and P. J. Kellogg, Phys. Rev. **107**, 1372 (1957).
[c] L. Rosen and J. E. Brolley, in *Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958*, (United Nations, Geneva, 1958), Vol. 14, p. 116.
[d] A. E. Glassgold, W. B. Cheston, M. L. Stein, S. B. Schuldt, and G. W. Erickson, Phys. Rev. **106**, 1207 (1958).
[e] Analysis was carried out by the authors.
[f] M. Takeda, J. Phys. Soc. Japan **15**, 557 (1960).
[g] I. Slaus and W. P. Alford, Phys. Rev. **114**, 1054 (1959).
[h] See reference 12.
[i] G. Igo, W. Lorenz, and U. Schmidt-Rohr, Phys. Rev. **121**, 1423 (1961).
[j] R. Wilson and J. Wesolowski (private communication by R. Bercaw).
[k] M. A. Melkanoff, *Proceedings of the International Conference on the Nuclear Optical Model, Florida State University Studies, No. 32* (Rose Printing Company, Tallahassee, Florida, 1959), p. 207.

As for stripping, two sets of parameters, differing primarily in V_d, were found to give acceptable results for the angular distribution (since R_{0d} is kept constant, this of course, is *not* the usual VR^2 ambiguity). However, the values obtained cannot be considered too reliable since the agreement with the experimental data at large angles is poor, and because the high imaginary deuteron potentials found necessary result in an insensitivity of the calculated angular distribution to variations in V_d. It is to be noted that for oxygen, as well as for other reactions, the values of W_d obtained from elastic-scattering data are generally much larger than those derived from stripping measurements. However, calculations made by Robson[8] for the deuteron elastic-scattering differential cross section for carbon indicate that inclusion of the spin-orbit interaction has the effect of raising the cross-section minima relative to the maxima, thus making high values of W_d (which produce the same general effect) unnecessary. In addition, inclusion of the spin-orbit interaction increases the cross section at large angles, thereby overcoming another difficulty encountered in non-spin-orbit deuteron elastic-scattering calculations. Hence, detailed comparisons between deuteron elastic scattering and stripping parameters—at least, for light nuclei—do not seem worthwhile unless a spin-orbit interaction is included in the elastic scattering calculations.

A somewhat large value for the proton real potential, $V_p=63$ MeV, appears necessary to obtain a good fit to the $E_d=7.73$-MeV data. This is higher than is found

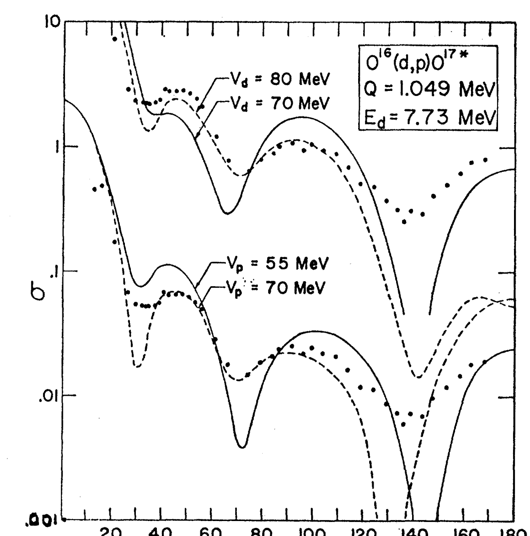

FIG. 4. Angular distributions for the $O^{16}(d,p)O^{17*}$, $L_N=0$ reaction showing the inferior agreement obtained using values of V_p and V_d different from those listed in Table II. The upper curves were calculated for V_d's near those found for low energy data. The values of the other parameters are $V_p=58$ MeV, $W_p=8$ MeV, $W_d=16$ MeV. The values of V_d, W_p, and W_d for the lower curves are the same as in Table II.

[8] D. Robson, Nucl. Phys. **22**, 34 (1961).

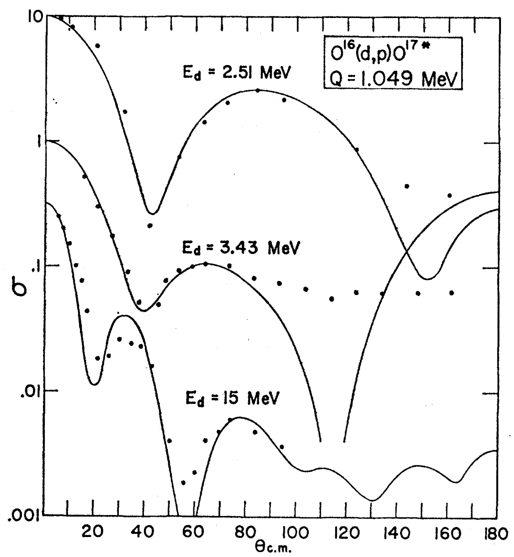

FIG. 5. Comparison of experimental and theoretical angular distributions for the $O^{16}(d,p)O^{17*}$, $L_N=0$ reaction for $E_d=2.51$, 3.43, and 15 MeV. Large values for V_d have been used. The parameters are listed in Table II.

for the $Si^{28}(d,p)Si^{28}$ reaction, for example, and is also greater than the values determined in a number of cases for elastic scattering of protons from light nuclei (Table III). The angular distributions for oxygen for $V_p=55$ and 70 MeV are shown in Fig. 4.

At lower energies for oxygen V_p is less critical. Figure 6 shows that for $E_d=2.51$ MeV any potential between 55 and 70 MeV yields the correct height and position of the second peak at $\theta=85°$, provided the imaginary potentials are suitably adjusted. Table IV

TABLE IV. Parameters for Figs. 6, 9, 10, and 17.

Figure	Reaction	L_N	E_d (MeV)	V_p (MeV)	V_d (MeV)	W_p (MeV)	W_d (MeV)	$\sigma_{calc}/\sigma_{exp}$
6	$O^{16}(d,p)O^{17*}$	0	2.51	50	78	0.6	1.2	0.465
				55		2.2	4.4	0.755
				60		2.8	5.6	0.827
				65		3.25	6.5	0.87
				70		4.05	8.1	0.849
				75		6.5	13	0.685
9	$Si^{28}(d,p)Si^{29}$	0	4	53	100	5	8	
			6.2	51	70	3	8	
				50	88	3	9	
10	$Si^{28}(d,p)Si^{29}$	0	8	51	75	2	6	
17	$O^{16}(d,p)O^{17}$	2	2.65	66	117.5	0.25	0.5	5.1
						0.5	1	3.02
						1	2	1.55
						2	4	0.914
						4	8	0.664

lists the parameters used. An excellent over-all fit can be obtained for $V_p=66.5$ MeV, in good agreement with $V_p=63$ MeV found for $E_d=7.73$ MeV, and also with the value of 66 MeV determined from a study of the stripping reaction leading to the ground state of O^{17}. Since in the present case the results were relatively insensitive to variations in W_p, its value, somewhat arbitrarily, was kept at half the value of W_d. Above $E_d=3$ MeV the choice of W_p becomes more restricted, as does that of V_p.

An extensive investigation was made for $E_d=7.73$ MeV to determine to what extent the results obtained for the angular distribution depend on R_p and R_d. It is found that the best value for V_p for a given R_p is very nearly independent of R_d, and that the best value of V_d for a given R_d is very nearly independent of R_p. The following relations are obtained:

$$V_pR_{0p}^2=31.1+52R_{0p},$$
$$V_dR_{0d}^2=56.5+97R_{0d},$$

where the potentials are in MeV and the radii are in F. It is also found that for best results the diffuseness parameters a_p and a_d should be increased by approximately 0.03 and 0.05 F whenever R_{0p} and R_{0d}, respectively, are decreased by 0.1 F.

Since it is relatively difficult to excite O^{16} (an excitation energy of at least 6.05 MeV is necessary), the strong low-lying levels of O^{17} should be particularly pure single-particle states. The ratio $\sigma_{calc}/\sigma_{exp}$ of the distorted-wave Born approximation for the absolute differential cross section to the experimental absolute differential cross section calculated at the principal peak should then be expected to have a value equal to one or, at most, slightly greater than one. The actual ratios obtained for the various energies (Table II) are (with a single exception) less than one. Futhermore, the values vary considerably from one energy to the next.

One way to increase this ratio is to increase R_N or a_N, thereby decreasing the magnitude of the interior oscillations of the neutron wave function. As a result, the neutron wave function normalization factor, occurring in the denominator of the expression for the absolute differential cross section, becomes smaller and, hence, the cross section larger. Thus, a neutron potential of greater range than used here, or more tapered, may be indicated.

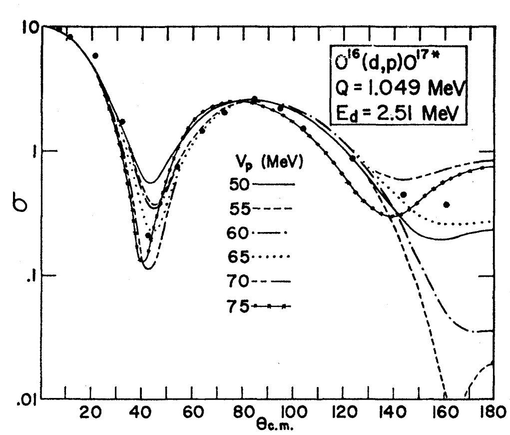

FIG. 6. Angular distributions for the $O^{16}(d,p)O^{17*}$, $L_N=0$ reaction for a wide range of values of V_p at $E_d=2.51$ MeV. The values of W_p and W_d have been optimized for each curve. The parameters are listed in Table IV.

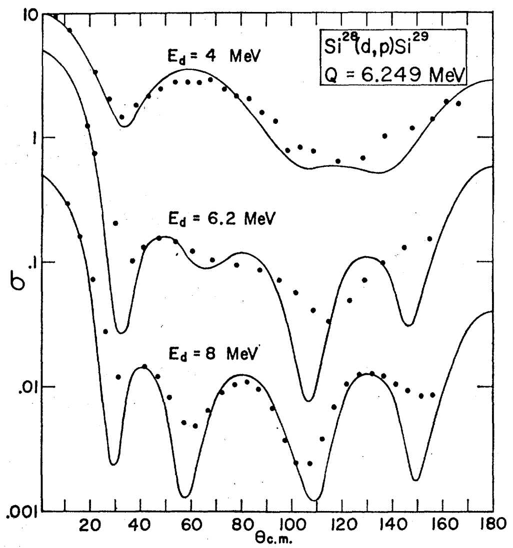

FIG. 7. Comparison of experimental and theoretical angular distributions for the $Si^{28}(d,p)Si^{29}$, $L_N=0$ reaction for $E_d=4$, 6.2, and 8 MeV. The parameters are listed in Table II.

A considerable amount of resonance structure is present in the excitation function of the $O^{16}(d,p)O^{17*}$ reaction. The total cross section appears to have a resonance maximum for $E_d=1.6$ MeV and a resonance minimum for $E_d=2$ MeV, although there is some uncertainty as to the precise energy values.[9] That resonances affect the experimental angular distributions for this reaction is evident from the fact that for $E_d=4.11$ MeV the cross section at the second maximum relative to that at the first maximum is only about half that for $E_d=3.945$ MeV.[10] In view of this the obtainability of reasonable fits to the low-energy $O^{16}(d,p)O^{17*}$ data may be due at least in part to the relatively simple shapes of the angular distributions. The principal effect of the resonances is to change the height of the second peak with respect to the main peak at 0°, and this behavior can be duplicated by proper variation of the imaginary potentials. As will be seen, a similar situation occurs for the $O^{16}(d,p)O^{17}$ ground-state reaction.

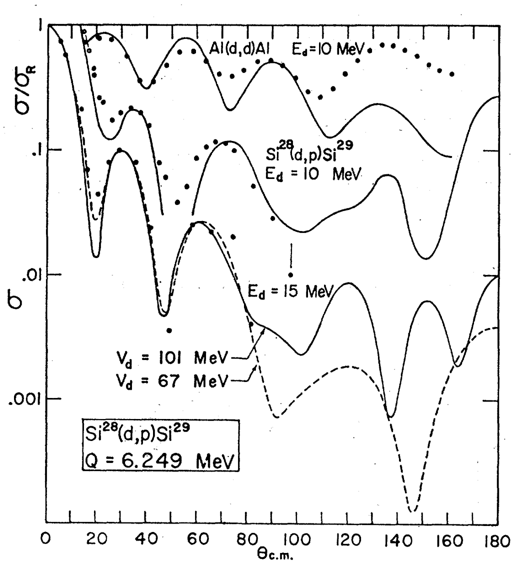

FIG. 8. Top: Comparison of experimental and theoretical angular distributions for elastic scattering of deuterons on aluminum at a bombarding energy of 10.1 MeV. The parameters are listed in Table III (second set). Middle and bottom: Comparison of experimental and theoretical angular distributions for the $Si^{28}(d,p)Si^{29}$, $L_N=0$ reaction for $E_d=10$ and 15 MeV. The parameters are listed in Table II.

$Si^{28}(d,p)Si^{29}$ (ground)

Results for the $Si^{28}(d,p)Si^{29}$ ground state, $L_N=0$ reaction are shown in Figs. 7 and 8 for $E_d=4$, 6.2, 8, 10,

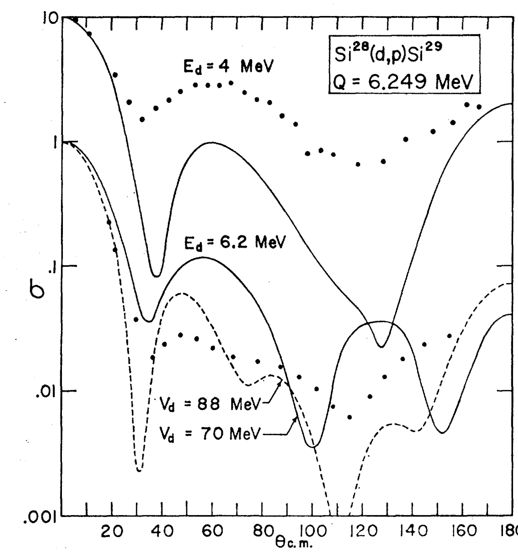

FIG. 9. Angular distributions for the $Si^{28}(d,p)Si^{29}$, $L_N=0$ reaction showing the inferior agreement obtained for $E_d=4$ MeV for V_d's having values near those found at higher energies, and also the inferior agreement for $E_d=6.2$ MeV when values of V_d are used which are intermediate between the optimum values for $E_d=4$ MeV and $E_d=6.2$ MeV. The parameters are listed in Table IV.

and 15 MeV. In general, the agreement with experiment is good, though, as for the $O^{16}(d,p)O^{17*}$ reaction near the Coulomb barrier an increase in V_d with E_d appears to be necessary, a rise of 30 MeV from $E_d=4$ MeV to $E_d=6.2$ MeV being obtained. It would be of some interest to have additional angular distribution measurements in this range.

The results of using a low value of V_d for 6.2 and 8 MeV, and of using a high value for 4 MeV, are shown in Figs. 9 and 10. The fits are seen to be quite unsatisfactory.

It will be noted that V_p is about 50 MeV for all the

[9] J. C. Grosskreutz, Phys. Rev. **101**, 706 (1961).

[10] E. Baumgartner and H. W. Fulbright, Phys. Rev. **107**, 219 (1957).

bombarding energies, appreciably less than $V_p=64$ MeV obtained for the $O^{16}(d,p)O^{17*}$ reaction for $E_d=1.05$ to 7.73 MeV. The higher value for V_p was tried for silicon, but was found to give inferior results.

The average of $\sigma_{calc}/\sigma_{exp}$ for the silicon ground-state reaction is roughly twice that for the first excited state oxygen reaction. Such a result is not unexpected, since Si^{28} is more readily excited than O^{16}, and, hence, the effect of admixtures of states is more important.

$Mg^{24}(d,p)Mg^{25*}$ (0.58 MeV)

Results for the $Mg^{24}(d,p)Mg^{25*}$ 0.58-MeV level, $L_N=0$ reaction for $E_d=3.9$ and 10 MeV are presented in Fig. 11. The values of V_d and V_p obtained are seen to be in rough agreement with those for the $Si^{28}(d,p)Si^{29}$ reaction at comparable energies.

$C^{12}(d,p)C^{13*}$ (3.09 MeV)

Calculated angular distributions for the $C^{12}(d,p)C^{13*}$ 3.09-MeV level, $L_N=0$ reaction are shown in Fig. 12

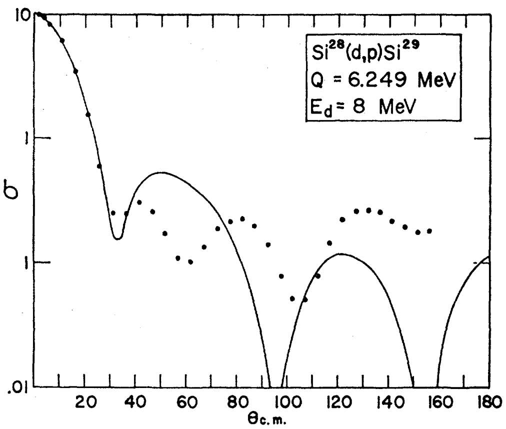

FIG. 10. Angular distributions for the $Si^{28}(d,p)Si^{29}$, $L_N=0$ reaction showing the inferior agreement obtained for $E_d=8$ MeV when a value of V_d is used which is 20 MeV smaller than the value yielding the fit shown in Fig. 7. The parameters are listed in Table IV.

for $E_d=2.889$ and 9 MeV. The fits obtained are fair, though the minima tend to be low, and the calculated principal maximum for $E_d=9$ MeV is somewhat broad.

An appreciable increase in V_d, similar to that observed for the $O^{16}(d,p)O^{17*}$, $Mg^{24}(d,p)Mg^{25*}$, and the $Si^{28}(d,p)Si^{29}$ reactions, is noted as the bombarding energy is raised from $E_d=2.889$ to 9 MeV. The value obtained for V_d for $E_d=2.889$ MeV agrees with the low-energy results found for the other $L_N=0$ reactions investigated. In the present instance, however, V_d is determined rather more sensitively since the extended horizontal portion of the back-angle distribution can only be obtained for values of V_d which are nearly the same for a wide variation of V_p. The height of the back-angle cross section relative to that for the forward peak can

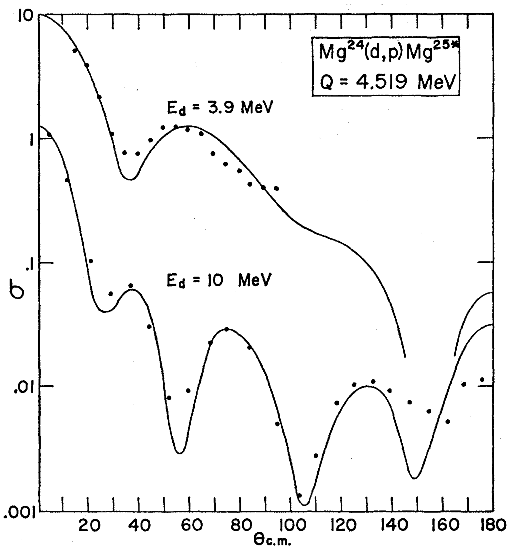

FIG. 11. Comparison of experimental and theoretical angular distributions for the $Mg^{24}(d,p)Mg^{25*}$, $L_N=0$ reaction for $E_d=3.9$ and 10 MeV. The parameters are listed in Table II.

be adjusted by varying W_p and W_d, the ratio becoming maximum for $W_p=0$, $W_d=0$. It is found that even under the most favorable parametric conditions the calculated cross section at large angles is unacceptably small unless V_p is chosen to be at least 60 MeV (the actual best-fit value obtained is $V_p=75$ MeV). Thus, as for oxygen, the optimum value for V_p is larger than for elastic scattering. A large V_p also appears to be indicated in a few studies (the results of which are not shown here) of higher energy data (8.2 to 13.3 MeV).

The ratio of the calculated to experimental absolute cross sections is roughly the same as that for the $O^{16}(d,p)O^{17*}$ reaction, indicating a single-particle character for the 3.09-MeV state of C^{13}.

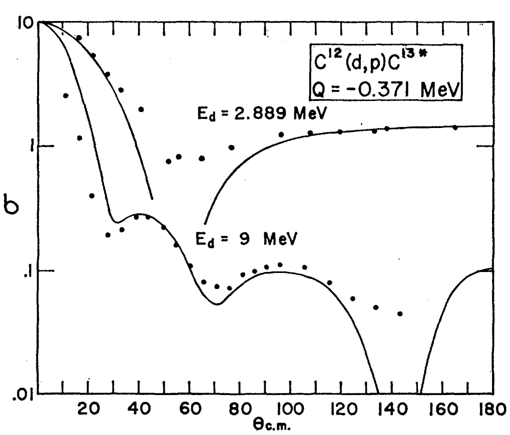

FIG. 12. Comparison of experimental and theoretical angular distributions for the $C^{12}(d,p)C^{13*}$, $L_N=0$ reaction for $E_d=2.889$ and 9 MeV. The parameters are listed in Table II.

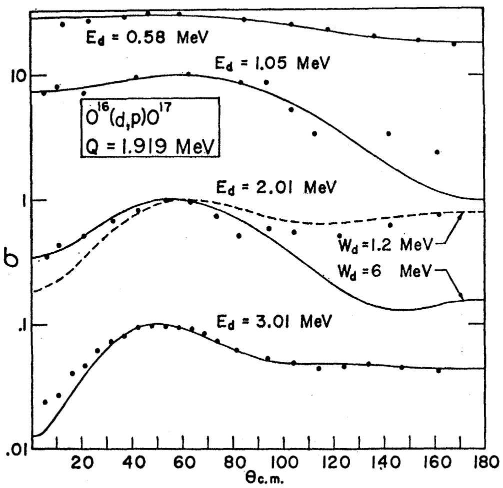

FIG. 13. Comparison of experimental and theoretical angular distributions for the $O^{16}(d,p)O^{17}$, $L_N=2$ reaction for $E_d=0.58$, 1.05, 2.01, and 3.01 MeV. The parameters are given in Table II, the dashed curve corresponding to the first set listed for $E_d=2.01$ MeV.

RESULTS FOR $L_N=2$ REACTIONS

$O^{16}(d,p)O^{17}$ (ground)

Angular distributions for the $O^{16}(d,p)O^{17}$ ground state, $L_N=2$ reaction for $E_d=0.58$, 1.05, 2.01, 3.01, 4.11, 7.73, and 15 MeV are presented in Figs. 13 and 14. It is seen that there is a persistent disagreement between the calculated results and the experimental data at larger angles, though for small angles the fits generally are good. The secondary peak at 60° for $E_d=15$ MeV is not well reproduced. The curve shown in Fig. 14 is the best obtained.

The values of V_d (Table II) are approximately 73 MeV for $E_d=0.58$ to 7.73 MeV, in agreement with the $E_d=1.05$ to 2.01 MeV results for the excited-state reaction, but do not exhibit the large variation with E_d observed for the $L_N=0$ reactions. The results for V_p for the ground and for the first excited-state reactions are seen to be also in good accord over a wide range of bombarding energies. The value $V_d=80$ MeV for $E_d=15$ MeV is 7 MeV higher than for $E_d=4.11$ MeV. Using a more shallow potential results in a peak at 0°, in disagreement with the data. The effect of using $V_d=100$ MeV, obtained for the first excited-state reaction at $E_d=7.73$ MeV, for the ground-state reaction at the same bombarding energy is exhibited in Fig. 15. It is seen that the fit is rather poor. As in previous cases, however, a still higher set of values for V_d is found to give results (Fig. 16) similar to those obtained using the best-fit set.

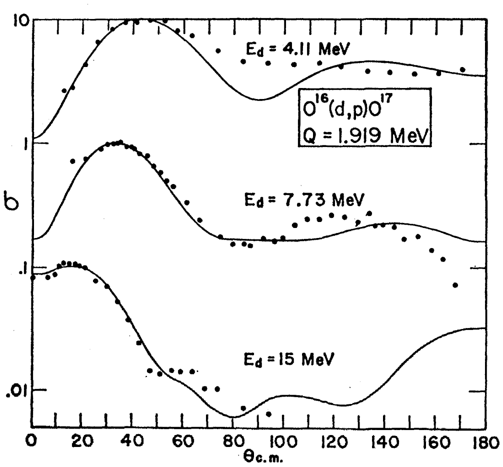

FIG. 14. Comparison of experimental and theoretical angular distributions for the $O^{16}(d,p)O^{17}$, $L_N=2$ reaction for $E_d=4.11$, 7.73, and 15 MeV. The parameters are listed in Table II.

Two curves are shown in Fig. 13 for $E_d=2.01$ MeV. One, calculated using very small values for W_p and W_d, has the large backward cross section required by the data, though the absolute cross section is anomalously large. The other is obtained using more usual values for the imaginary potentials, but disagrees with the angular distribution at large angles, though the absolute cross section is more reasonable.

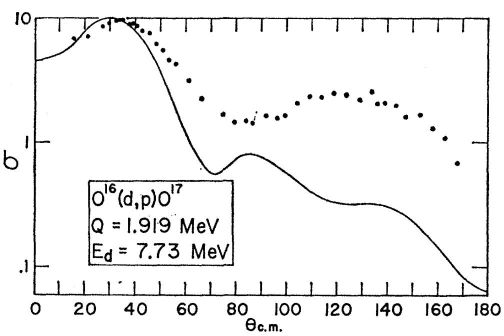

FIG. 15. Angular distributions for the $O^{16}(d,p)O^{17}$, $L_N=2$ reaction for $E_d=7.73$ MeV showing the inferior agreement obtained using a value of V_d which yields good results for the $O^{16}(d,p)O^{17*}$, $L_N=0$ reaction at the same energy.

The same effects are present for $E_d=2.65$ MeV. In Fig. 17 calculated angular distributions are presented for a wide range of imaginary potentials with $W_d=2W_p$. The corresponding absolute cross sections are given in Table IV, and are seen to be very sensitive to the values of the imaginary potentials. At higher bombarding energies this sensitivity diminishes.

It is possible that the anomalously high backward cross sections observed for this reaction near $E_d=2.5$ MeV, as well as similar effects for other reactions, may be due to fluctuations in the level density of the compound nucleus. Alternatively, the large backward yields may be the result of exchange stripping. Nagarajan and Banerjee,[11] using plane waves, obtained good agree-

[11] M. A. Nagarajan and M. K. Banerjee, Nucl. Phys. **17**, 341 (1960).

ment with the $E_d=3.49$ MeV $O^{16}(d,p)O^{17}$ data by including an exchange-stripping amplitude in the calculation.

$Mg^{24}(d,p)Mg^{25}$ (ground)

In Fig. 18 are presented angular distributions for the $Mg^{24}(d,p)Mg^{25}$ ground state, $L_N=2$ reaction for $E_d=10$ and 14.8 MeV. It is seen that for $E_d=10$ MeV there is an almost perfect mismatch between calculational and experimental structure at larger angles, a result also obtained by Buck and Hodgson.[12]

In the calculations, V_p and V_d were varied in 10-MeV intervals with V_p ranging from 35 to 65 MeV and V_d from 50 to 120 MeV. All possible combinations of V_p and V_d were considered. Possible fits for $E_d=10$ MeV could be found only for the rather extreme values $V_p=35$ MeV and $V_d=105$ MeV. The best-fit curve shown in Fig. 18 was obtained by further varying W_p and W_d. The $E_d=14.8$ MeV experimental distribution is an example of an $L_N=2$ reaction which has an anomalous peak at 0° which, nevertheless, can be well reproduced by distorted wave calculations.

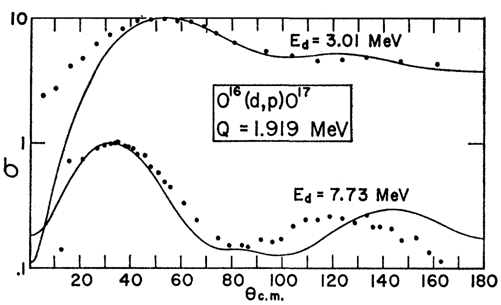

FIG. 16. Comparison of experimental and theoretical angular distributions for the $O^{16}(d,p)O^{17}$, $L_N=2$ reaction for $E_d=3.01$ and 7.73 MeV. Large values for V_d have been used. The parameters are listed in Table II.

The unacceptable distributions determined for $E_d=10$ MeV serve as a warning that caution must be exercised in applying the distorted-wave Born approximation with optical-model potentials to stripping reactions with light nuclei, even at fairly high bombarding energies. Trial calculations for a comparable case, the $Si^{28}(d,p)$-Si^{29*} 1.28-MeV level, $L_N=2$ reaction at $E_d=8$ MeV, also gave poor results (not shown) for angles larger than 70°.

$S^{32}(d,p)S^{33}$ (ground)

Considerable effort to fit the available $E_d=4$ MeV data for the $S^{32}(d,p)S^{33}$ ground state, $L_N=2$ reaction failed. The best over-all result obtained is shown in Fig. 18.

[12] B. Buck and P. E. Hodgson, in *Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961*, edited by J. B. Birks (Heywood and Company Ltd., London, 1961), p. 443.

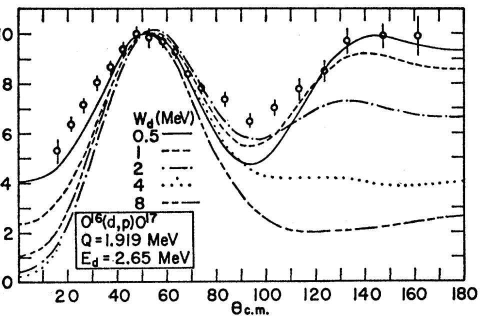

FIG. 17. Angular distributions for the $O^{16}(d,p)O^{17}$, $L_N=2$ reaction at $E_d=2.65$ MeV, demonstrating that the use of anomalously small values for W_p and W_d yields a high backward angular distribution in agreement with experiment. The parameters are listed in Table IV.

RESULTS FOR $L_N=1$ REACTIONS

$C^{12}(d,p)C^{13}$ and $C^{12}(d,n)N^{13}$ (ground)

Relative differential cross sections for the $C^{12}(d,p)C^{13}$ ground state, $L_N=1$ reaction at $E_d=9$ MeV and for the $C^{12}(d,n)N^{13}$ ground state, $L_p=1$ reaction at $E_d=2.75$ MeV are presented in Fig. 19. The fit is good in the former case but only fair in the latter. For the 9 MeV (d,p) reaction it is found that smaller than usual values for the diffuseness parameters, namely, $a_p=0.4$ F and $a_d=0.6$ F, improve the results considerably. Since it is apparent from the excitation curves and angular distributions at low energies for these reactions[13,14] that compound nucleus effects play a dominant role, the application of the distorted-wave Born approximation here must again be viewed with caution.

Calculations for the (d,p) reaction for several bom-

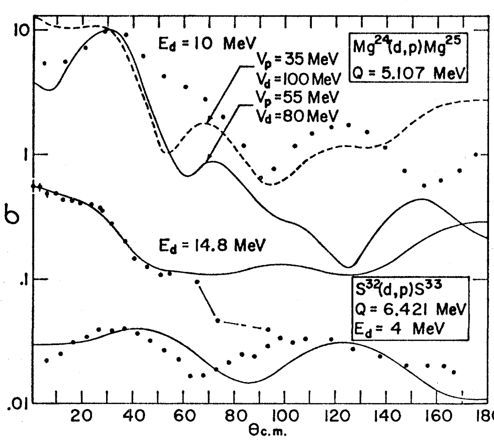

FIG. 18. Comparison of experimental and theoretical angular distributions for (top and middle) the $Mg^{24}(d,p)Mg^{25}$, $L_N=2$ reaction for $E_d=10$ and 14.8 MeV; and (bottom) the $S^{32}(d,p)S^{33}$, $L_N=2$ reaction at $E_d=4$ MeV. The parameters are listed in Table II.

[13] M. T. McEllistrem, K. W. Jones, Ren Chiba, R. A. Douglas, D. F. Herring, and E. A. Silverstein, Phys. Rev. **104**, 1008 (1956).

[14] A. Elwyn, J. V. Kane, S. Ofer, and D. H. Wilkinson, Phys. Rev. **116**, 1490 (1959).

barding energies below 4.5 MeV yield poor results (not shown). From the imperfect evidence obtained, V_d seems to increase with E_d.

$Be^9(d,p)Be^{10}$ (ground)

In calculating the angular distributions for the $Be^9(d,p)Be^{10}$ ground state, $L_N=1$ reaction at $E_d=3.6$ MeV, V_p and V_d were varied in 10-MeV steps for V_p between 50 and 70 MeV, and V_d between 50 and 110 MeV. All combinations of V_p and V_d were considered. In all cases results similar to those shown in Fig. 19 were obtained. It is clear that the calculated and experimental positions of the backward peak disagree markedly.

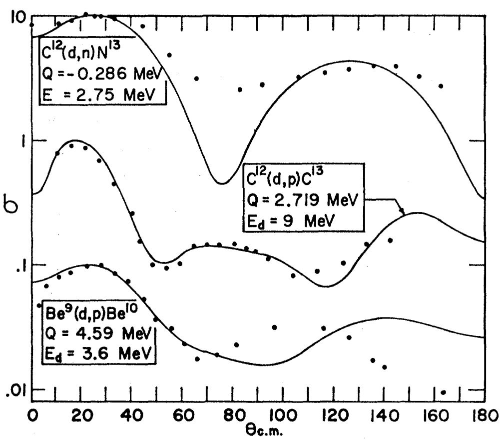

FIG. 19. Comparison of experimental and theoretical angular distributions for (top) the $C^{12}(d,n)N^{13}$, $L_p=1$ reaction at $E_d=2.75$ MeV, (middle) the $C^{12}(d,p)C^{13}$, $L_N=1$ reaction at $E_d=9$ MeV, and (bottom) the $Be^9(d,p)Be^{10}$, $L_N=1$ reaction at $E_d=3.6$ MeV. The parameters are listed in Table II.

$B^{10}(d,p)B^{11}$ (ground)

Calculations of the angular distribution for the $B^{10}(d,p)B^{11}$ ground state, $L_N=1$ reaction for $E_d=15.5$ and 21.5 MeV have been performed by Zeidman *et al.*[15] with good results. It was thought of interest to confirm and extend this work, using present procedures.

For the high-bombarding energies involved here it is found that the calculated positions of the peaks agree with the experimental data for a wide range of optical-model parameters. However, since only the relative heights of the various peaks change appreciably in varying V_p, V_d, W_p, W_d, a_p, and a_d, it is difficult to determine a best-fit set of values. It also appears that the effect of varying a particular parameter depends appreciably on the values of the other parameters.

The calculated angular distributions for E_d ranging from 8.2 to 28 MeV are presented in Fig. 20, and the corresponding values for V_p and V_d are listed in Table II.

[15] B. Zeidman, J. L. Yntema, and G. R. Satchler, in *Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961* (Heywood and Company Ltd., London, 1961), p. 515.

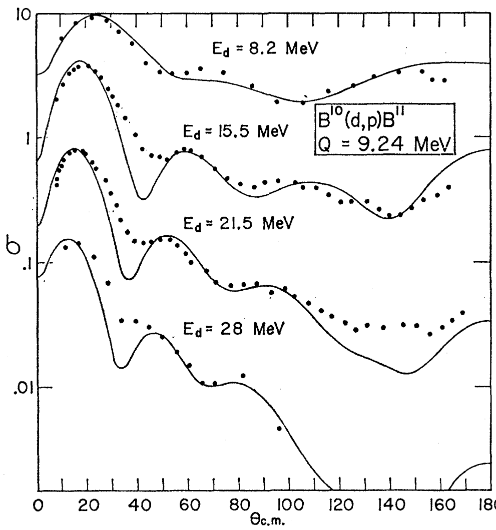

FIG. 20. Comparison of experimental and theoretical angular distributions for the $B^{10}(d,p)B^{11}$, $L_N=1$ reaction for $E_d=8.2$, 15.5, 21.5, and 28 MeV. The parameters are listed in Table II.

An attempt has been made to maintain a systematic variation with energy of these parameters. The increase in V_d from $E_d=15.5$ to 21.5 MeV proves necessary in order to lower the second peak to the correct height. Varying the other parameters does not give a sufficiently large effect. As for the $C^{12}(d,p)C^{13}$ reaction, smaller than usual values for the diffuseness, $a_p=0.4$ F and $a_d=0.6$ F, are found to yield slightly better results for the angular position of the main peak for $E_d=15.5$ MeV.

For $E_d=28$ MeV several significantly different sets of parameters have been obtained which yield results as satisfactory as those presented in Fig. 20. One such set is the following: $a_p=0.5$ F, $a_d=0.7$ F, $V_p=50$ MeV, $V_d=95$ MeV, $W_p=10$ MeV, $W_d=20$ MeV.

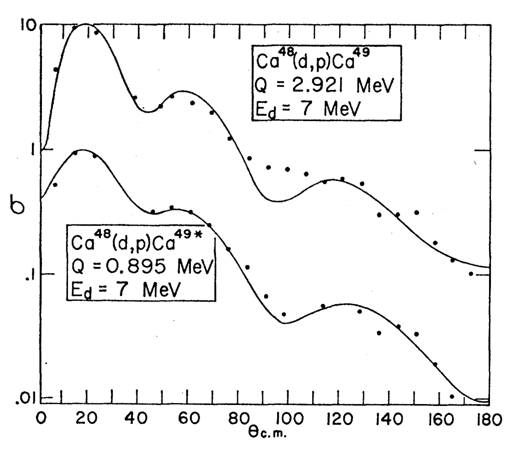

FIG. 21. Comparison of experimental and theoretical angular distributions for (top) the $Ca^{48}(d,p)Ca^{49}$, $L_N=1$ reaction at $E_d=7$ MeV, and (bottom) the $Ca^{48}(d,p)Ca^{49*}$, $L_N=1$ reaction at $E_d=7$ MeV. The parameters are listed in Table II.

TABLE V. Parameters for Figs. 22 and 23.

Figure	Reaction	L_N	R_{0p} (F)	R_{0d} (F)	a_p (F)	a_d (F)	V_p (MeV)	V_d (MeV)	W_p (MeV)	W_d (MeV)	$\sigma_{calc}/\sigma_{exp}$
22	$O^{16}(d,p)O^{17}$	2	1.5	1.7	0.4	0.55	47	43	5.25	10.5	0.705
	$O^{16}(d,p)O^{17*}$	0	1.5	1.7	0.35	0.35	47	72	6	12	0.421
	$O^{16}(d,p)O^{17*}$	0	1.1	1.2	0.6	0.85	73	120	8	16	0.286
23	$O^{16}(d,p)O^{17}$	2	1.25	1.4	0.5	0.7	65	75	5	10	0.693
	$O^{16}(d,p)O^{17*}$	0	1.25	1.4	0.5	0.7	68	105	6	12	0.382

$Ca^{48}(d,p)Ca^{49}$ (ground and 2.026 MeV)

Angular distributions for the $Ca^{48}(d,p)Ca^{49}$ ground state and 2.026-MeV level, $L_N=1$ reactions are presented in Fig. 21 for $E_d=7$ MeV. Good fits are obtained using optical parameters in agreement with the values found for the heavier nuclei (remembering the VR^2 ambiguity). Of course, since Ca^{48} has filled proton and neutron shells and is a fairly heavy nucleus, it is expected to be a particularly favorable target.

FURTHER INVESTIGATIONS

The $O^{16}(d,p)O^{17}$ ground and first excited-state reactions at $E_d=7.73$ MeV were selected in an attempt to determine whether improved results for angular distributions could be obtained by varying R_{0p}, R_{0d}, a_p, and a_d (which generally were kept constant in the course of the previously described calculations). In this manner it was hoped to examine the degree to which "best-fit" parameters are unique.

The procedure used was to choose relatively large radii ($R_{0p}=1.5$ F, $R_{0d}=1.7$ F), or comparatively small radii ($R_{0p}=1.1$ F, $R_{0d}=1.2$ F), and observe the effect of varying other parameters, particularly a_p and a_d. Figure 22 (also Table V) shows that good results are obtained using the larger radii for reactions leading to the ground and first excited states of O^{17}. A fair fit is also found for the first excited state of oxygen using the smaller radii (Fig. 22), though not for the ground state.

Varying the real and imaginary potentials while using the standard radii ($R_{0p}=1.25$ F, $R_{0d}=1.4$ F) and keeping both of the diffuseness parameters either large ($a_p=0.7$ F, $a_d=0.9$ F) or small ($a_p=0.25$ F, $a_d=0.4$ F) yielded results inferior to those already obtained for the ground- and first excited-state oxygen reactions. However, it was discovered for the $E_d=7.73$ MeV, $O^{16}(d,p)$-O^{17*} reaction—and subsequently for the $E_d=8$ MeV, $Si^{28}(d,p)Si^{29}$; $E_d=10$ MeV, $Mg^{24}(d,p)Mg^{25*}$; and the $E_d=9$ MeV, $C^{12}(d,p)C^{13*}$ reactions (all $L_N=0$)—that good agreement with experiment is obtained using $V_d=85$ MeV (10 to 20 MeV lower than the values listed in Table II) provided, however, that a_p is increased from 0.5 to 0.7 F (Table VI). The latter figure is appreciably higher than any of the elastic scattering values of Table III. Nor do the parameters of Table VI yield satisfactory results for the angular distribution when applied to the $E_d=7.73$ MeV, $O^{16}(d,p)O^{17}$ ground state or the $E_d=10$ MeV, $Mg^{24}(d,p)Mg^{25}$ ground-state reactions. In agreement with previous results (Table II) it appears that the (d,p) stripping reactions leading to the ground and first excited states of O^{17} and of Mg^{25} yield quite different sets of optical-model parameters.

TABLE VI. Alternate parameters.

Reaction	L_N	E_d (MeV)	a_p (F)	a_d (F)	V_p (MeV)	V_d (MeV)	W_p (MeV)	W_d (MeV)
$C^{12}(d,p)C^{13*}$	0	9	0.7	0.7	60	85	9	18
$O^{16}(d,p)O^{17*}$	0	7.73	0.7	0.7	55	85	8	16
$Mg^{24}(d,p)Mg^{25*}$	0	10	0.7	0.7	48	85	4	12.3
$Si^{28}(d,p)Si^{29}$	0	8	0.7	0.7	50	85	2	10

In another series of calculations surface-peaked imaginary potential form factors for the proton and deuteron were used. The real and imaginary form factors were chosen to be of the polynomial segment form labeled F_R and F_{so} described in a previous article.[1] Results clearly superior to those for volume absorption for the $O^{16}(d,p)O^{17}$ ground-state reaction, and at least as good

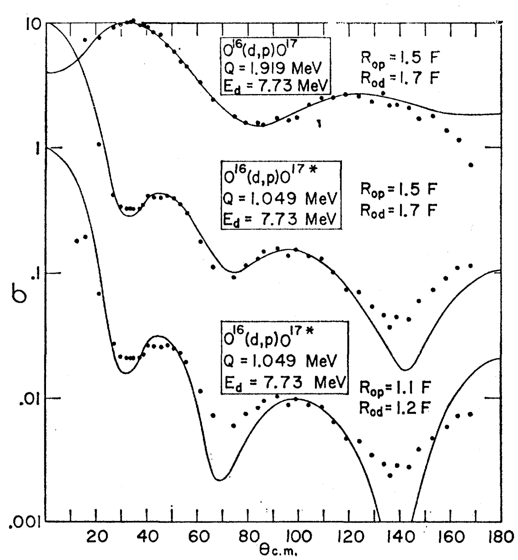

FIG. 22. Angular distributions (top) for the $O^{16}(d,p)O^{17}$, $L_N=2$ reaction at $E_d=7.73$ MeV, using large values of R_{0p} and R_{0d}, (middle) for the $O^{16}(d,p)O^{17*}$, $L_N=0$ reaction at $E_d=7.73$ MeV, and (bottom) for the $O^{16}(d,p)O^{17*}$, $L_N=0$ reaction at $E_d=7.73$ MeV, using small values of R_{0p} and R_{0d}. The parameters are listed in Table V.

as the volume absorption ones for the first excited-state reaction, are obtained (Fig. 23 and Table V).

The effect on the cross section of including spin-orbit interactions in the deuteron and proton channels for $j_N=\frac{1}{2}$ reactions has also been investigated, and is illustrated in Fig. 24 for the $L_N=0$, $Si^{28}(d,p)Si^{29}$ and the $L_N=1$, $C^{12}(d,p)C^{13}$ reactions. The calculations are based on the formulas of Robson.[8] A surface spin-orbit potential of the derivative Woods-Saxon type is used. It is seen that striking improvement is obtained for silicon at the larger angles. An improved agreement is also noted for those $L_N=0$ cases for which measurements extend to large angles, namely, for the silicon reaction at $E_d=4$ and 8 MeV, the oxygen reaction at $E_d=3$ and 7.73 MeV, and the magnesium reaction at $E_d=10$ MeV. The improvement obtainable is such that the last minimum can be raised by the proper amount (except for oxygen at $E_d=7.73$ MeV, for which the magnitude of the effect is too small). At low energy—at $E_d=1.6$ MeV for oxygen, for example—the inclusion of spin-orbit potentials changes the angular distribution only by a small amount. Spin-orbit effects for the first excited-state $Ca^{48}(d,p)Ca^{49*}$ reaction at $E_d=7$ MeV are also rather small, and can be reproduced by variation of some of the other parameters. For the $C^{12}(d,p)C^{13}$ reaction at $E_d=9$ MeV inclusion of the spin-orbit interactions is seen to result in a somewhat better fit, though, unfortunately, the most noticeable effect is for angles outside the range of presently available data.

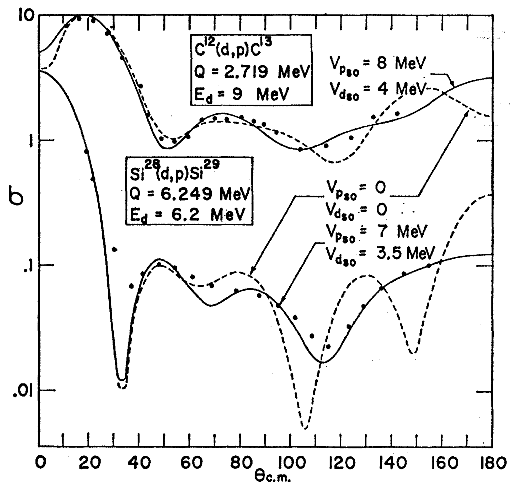

FIG. 24. Results illustrating the effect of including spin-orbit interactions in the proton and deuteron channels on the angular distributions. Top: The $C^{12}(d,p)C^{13}$, $L_N=1$ reaction, the non-spin-orbit parameters agreeing with those of Fig. 19. Bottom: The $Si^{28}(d,p)Si^{29}$, $L_N=0$ reaction with $V_p=50$ MeV, $V_d=100$ MeV, $W_p=3$ MeV, and $W_d=8$ MeV.

DISCUSSION

From the results presented it is apparent that there is a tendency for agreement between calculated and experimental angular distributions to diminish with increasing values of L_N. The same behavior is observable in the results of Buck and Hodgson[12] for the $L_N=0$ and $L_N=2$ (d,p) reactions on Mg^{24} at $E_d=10$ MeV, and of Tobocman and Gibbs[16] for the $L_N=1$ and $L_N=3$ (d,p) reactions on Ca^{40} at $E_d=4.13$ and 4.69 MeV. The effect also appears for heavier targets, as for the $Zn^{68}(d,p)Zn^{69}$ reactions[1] at $E_d=11.9$ MeV, where good agreement is obtained for the $L_N=1$ distribution, but difficulty is encountered for the $L_N=2$ and $L_N=4$ reactions. Since the parameters which were held constant in the present investigation (R_{0p}, R_{0d}, a_p, and a_d) were determined mainly from studies with $L_N=0$ reactions, it is possible that the diminishing agreement with increasing L_N observed is due to a dependence of these quantities on L_N.

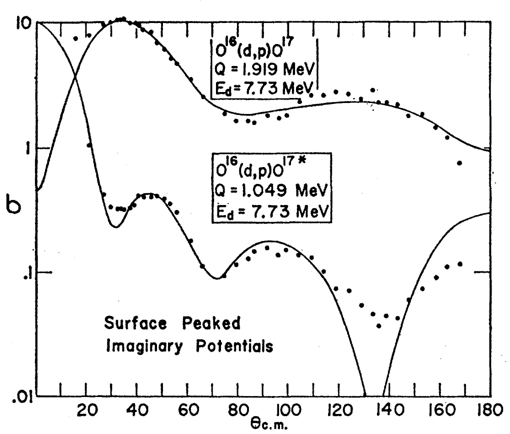

FIG. 23. Results obtained using surface-peaked form factors for W_p and W_d for (top) the $O^{16}(d,p)O^{17}$, $L_N=2$ reaction at $E_d=7.73$ MeV, and (bottom) the $O^{16}(d,p)O^{17*}$, $L_N=0$ reaction at $E_d=7.73$ MeV. The parameters are listed in Table V.

Conflicting evidence is obtained regarding the applicability of the distorted-wave Born approximation with optical-model potentials to stripping reactions for light nuclei. In many cases it is possible to attain fair agreement with the experimental data, though, as has been seen, there are a number of reactions for which this appears impossible using physically reasonable optical-model parameters. For the latter instances there is, of course, always the possibility that due to the complexity of the calculations a more extensive investigation will yield results which are more acceptable, though it is clear that the calculational procedures described here and in reference 1 are successful for the heavier target nuclei with $A\geq 48$ and, presumably, should be adequate for light nuclei as well.

As is apparent from Table III, there is considerable variation in the deuteron and proton elastic-scattering parameters for a number of light nuclei, though the averages of a_p and a_d are in satisfactory agreement with the stripping values of Table II. However, W_d

[16] W. Tobocman and W. R. Gibbs, Phys. Rev. **126**, 1076 (1962).

for elastic scattering is higher than for stripping except in the case of $B^{10}(d,p)B^{11}$ above $E_d = 15$ MeV. There is also a less marked tendency for W_p for elastic scattering to be greater than for stripping. The $L_N = 0$ silicon and magnesium reactions, as well as the 9 MeV, $L_N = 1$ carbon reaction, for which good results are obtained, in particular, seem to require values of W_p and W_d which are considerably smaller than for elastic scattering.

As previously mentioned, the high values of W_d found from elastic-scattering data result in an insensitivity of the angular distributions to variations in V_d, so that V_d is not accurately determined for elastic scattering. Nevertheless, there appears to be a real discrepancy between stripping and elastic-scattering results for V_d in many of the cases considered here. The real proton potential V_p is roughly the same for elastic scattering and stripping, except for the $L_N = 0$ and 2 oxygen reactions and the $L_N = 0$ carbon reaction, for which the stripping values are rather high.

In general, then, the agreement of stripping optical-model parameters with the elastic scattering values for light nuclei is poor in comparison with that obtained for heavy nuclei.[1] Of course, even for elastic scattering the optical-model analyses for light nuclei in many cases give unsatisfactory results.[17] It is also by no means certain that the two sets of parameters, particularly for light nuclei, must necessarily be the same. Indeed, there is a significant difference between elastic scattering and stripping mechanisms in that stripping occurs predominantly at the nuclear surface, so that a more restricted range of angular momenta for deuterons and protons is involved than for elastic scattering. Consequently, the scattering conditions are not quite the same in the two cases, and the effective optical-model potentials may well be different. For stripping there is an additional complication in that the initial and final nuclei are different. For light nuclei this difference may be significant.

We repeat our previously made observation (reference 1) that the results obtained using the distorted-wave Born approximation with optical potentials should be most appropriate for the heavier nuclei. Compound nucleus, exchange stripping, and mass-correction effects should be smaller, and the optical-model approximation better.

Nevertheless, as has been seen, a considerable degree of success has been obtained for light nuclei in many instances. The $B^{10}(d,p)B^{11}$ angular distributions from $E_d = 8.2$ to 28 MeV are fairly well reproduced using reasonable optical-model parameters. Consistent parameters for the $Si^{28}(d,p)Si^{29}$ reaction for E_d between 6.2 and 15 MeV, and for the $O^{16}(d,p)O^{17}$ reaction for E_d between 1.05 and 15 MeV have been obtained, although some of the fits for the latter reaction are inferior. The results for the two reactions on Ca^{48} at $E_d = 7$ MeV are also good. A forward peak for the $L_N = 2$, $Mg^{24}(d,p)Mg^{25}$ reaction at $E_d = 14.8$ MeV, not predicted by the simple plane-wave Butler stripping theory,[18] is well reproduced. The observed high-backward cross sections for the $O^{16}(d,p)O^{17}$ reaction also have been obtained, though it has proved necessary to use unusually small imaginary potentials which result in anomalously large absolute cross sections.

It is evident that V_p is not a smoothly varying function of A. For example, an appreciable decrease in this parameter occurs in going from O^{16} to Mg^{24}. However, except for the two $L_N = 0$, C^{12} reactions, nearly all the values fall in the range from 50 to 65 MeV. The variation in V_d is considerably greater, though it is to be noted that for higher bombarding energies the values of V_d are similar for the $L_N = 0$ reactions on C^{12}, O^{16}, Mg^{24}, and Si^{28}.

The poorest results obtained for the angular distribution are for the $L_N = 1$, $Be^9(d,p)Be^{10}$ and the low-bombarding energy $C^{12}(d,p)C^{13}$ reactions, and the $L_N = 2$, $Mg^{24}(d,p)Mg^{25}$, $Si^{28}(d,p)Si^{29*}$, and $S^{32}(d,p)S^{33}$ reactions [some of the $O^{16}(d,p)O^{17}$ data also were not well fitted]. For the $L_N = 0$ reactions on C^{12}, O^{16}, Mg^{24}, and Si^{28} a disturbing feature is the large increase in V_d with bombarding energy found in the neighborhood of the Coulomb barrier.

It is apparent that much additional work remains to be done in the study of stripping reactions involving light nuclei. Additional data, especially for targets of O^{16} and Si^{28}, would be most helpful. In this connection the importance of extending measurements over as wide an angular distribution as possible should be emphasized.

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[18] T. B. Sutler, Proc. Roy. Soc. (London) **A208**, 559 (1951).